

Mathematical model for the study and design of a solar dish collector with cavity receiver for its application in Stirling engines[†]

Ricardo Beltran^{1,*}, Nicolas Velazquez¹, Alma Cota Espericueta², Daniel Saucedo³ and Guillermo Perez¹

¹*Centro de Estudios de las Energías Renovables, Universidad Autónoma de Baja California, Blvd. Benito Juárez y Calle de la Normal s/n, Mexicali, Baja California 21280, México*

²*Instituto de Ciencias Biomédicas, Universidad Autónoma de Ciudad Juárez, Henri Dunant 4016, Zona Pronaf, Hermosillo, Sonora 32310, México*

³*Facultad de Ingeniería, Universidad Autónoma de Baja California, Blvd. Benito Juárez y Calle de la Normal s/n, Mexicali, Baja California 21280, México*

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Abstract

This paper presents a mathematical model that allows representing the optical behavior of a solar parabolic dish concentrator and the thermal performance of a cavity receiver. A procedure and a graphical method for the design of dish/cavity systems are proposed. A parametric study of the main geometric variables is performed and the influence of climate variables on the thermal behavior of the system coupled to a Stirling engine is analyzed. The model considers errors of solar collector, intercept factor, reflected and emitted radiation, conduction, and convection heat losses. For the validation of the model, the results obtained were compared with theoretical and experimental results reported in the literature. The calculation of the radiation losses, emitted and reflected from the receiver presented errors of up to 14%, and the average error for the rest of the thermal losses, interception factor and the absorber's temperature, was less than 3%. These results show that the proposed model can be used with sufficient certainty to design and optimize solar dish collectors with cavity receivers.

Keywords: Geometric-optical model; Heat losses, Parabolic dish collector; Stirling engine; Thermal model

1. Introduction

Currently, there is a growing global support for research and development of renewable energy technologies, particularly for power generation. Solar parabolic dish collectors and Stirling engines with cavity receivers are commonly considered for this purpose due to the high efficiency for converting solar radiation into mechanical energy. The study and design of a solar collector of this type, and of its cavity receiver, require solving a mathematical model that take into account the geometric, optical and thermal behavior of all components. With an adequate sizing, not only the useful energy produced on the solar device will meet the energy required for the process, but also the absorber temperature will be the needed for the operation of the Stirling engine.

Optical and thermal energy losses are commonly considered in these studies. The reflectance of the collector surface is included into the energy balance. The curvature irregularities on concentrators cause radiation spillage of the solar image produced at the focal zone. The solar radiation receiver posi-

tioned at the focal point of the concentrator will shade the reflective surface of the concentrator. The thermal losses taken into account are reflection and emission of radiation from the inner surfaces of the cavity receiver outwards through its aperture, convection also through the aperture, and the losses by conduction through the walls and receiver insulation [1].

This article focuses on the construction of a mathematical model that represents the operational performance of a concentrator with cavity receiver for its applications in Stirling engines. The purpose is to develop a designing tool for optimization and for quantifying the effect of changing the values of design parameters over any specific output behavior or the overall performance of the system; for instance, to compute the effect of such design parameters on the receiver efficiency and heat transfer rate to the Stirling engine. Several mathematical models have been published to represent the mechanisms of thermal and optical losses of an open cavity [1-3].

The use of solar concentrators requires tracking systems to maintain a zero incidence angle; however, irregularities on specular-reflective surfaces generate bigger solar image than the ideal size calculated theoretically. Harris and Lenz [1] analyzed the mechanisms of energy losses from the cavities and the influence of their geometry and the rim angle of the

*Corresponding author. Tel.: +52 686 5664150, Fax.: +52 686 5664150
E-mail address: rbeltran1@uabc.edu.mx

[†]Recommended by Associate Editor Ji Hwan Jeong

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concentrator on the temperature profile produced in the cavity. In such study the distribution of radiation coming from the concentrator was calculated over the focal plane using the error-cone optical model, considering the specular reflectance and the slope error. However, for actual operating conditions, the analysis must also include errors due to monitoring and tracking mechanical systems. Stine and Harrigan [4] considered optical errors and inaccuracies of tracking systems as typical errors in moving solar concentrators. In order to capture all of the solar radiation redirected from the collector into the receiver, the diameter of the cavity aperture must be enlarged to consider the optical errors; yet, this does lead to higher heat losses from the cavity walls. Seo et al. [5] calculated the energy losses in two receivers, one conical and one dome-shaped, using a multifaceted parabolic concentrator; the Monte Carlo method was used to calculate the losses by reflection of radiation, and the net radiation model to determine the radiation emitted from the inner surfaces of the cavity. This approach cannot be used for the analysis of a receiver with the absorber located only at the bottom of the cavity to supply the useful energy to a Stirling engine. Nepveu et al. [3] used a nodal method for calculating the energy losses by reflection, emission, convection, and conduction through the ceramic walls; for its solution, the solar image was characterized at different planes near the aperture, however this cannot be performed during the phase of design. Duffie and Beckman [6] developed a model to estimate the radiation reflected back from a non-covered cavity, taking into consideration the absorbance of the internal surfaces, and the area ratio between the inner walls and the aperture. Fraser [2] compared the radiation emitted by the cavity calculated from Stefan-Boltzmann equation considering the effective absorbance and the aperture area with that calculated by using view factors, finding similar results. The simplicity of the models used by Fraser [2] and Duffie and Beckman [6], make them attractive to perform dynamic simulations, whether hourly, daily, monthly or yearly, with little computing time. Nouanegue et al. [7] investigated the heat transfer in open cavities, by considering natural convection, conduction and radiation; for this study the heat flux reaching the inner surface facing the cavity aperture, was considered uniform. It was found that the surface emitted radiation modifies the flow pattern and temperature profiles. Sendhil and Reddy [8] investigated numerically natural convective losses in a receiver cavity, at a particular inclination, assuming an isothermal-hemispherical surface as absorber and a fuzzy focal solar dish. Sendhil and Reddy [9] investigated the performance of three receivers for a fuzzy focal solar dish concentrator and conclude that a modified cavity is the preferred receiver type. Prakash et al. [10] studied theoretically and experimentally, the profiles of air velocity and temperature within a solar cavity receiver, and proposed the term "critical air temperature gradient" and a dimensionless parameter related to the stagnation and convective zones. Their results confirmed the presence of significant temperature profiles within the cavity previously reported by

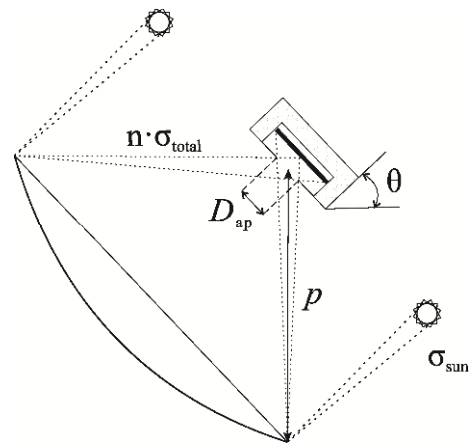


Fig. 1. Pathway of the solar beam in a solar parabolic dish collector with cavity receiver for a Stirling engine absorber.

Reinalter et al. [11]. More studies regarding heat transfer distributions within receiver cavities for Stirling engines must be conducted; it is necessary to evaluate the influence of distribution of solar radiation, geometry (deepness of the cavity) and stagnation zones on thermal efficiency.

The aim of this paper is to present a simple mathematical model that can be used for designing and optimization of thermal performance of solar parabolic dish collectors with cavity receivers. The purpose of the simplicity of the model is to require little computing time to perform dynamic simulations and to ease parametric studies and optimization. Since the dish/cavity solar collector systems are used in various applications requiring high temperature, such as Stirling engines, this paper proposes a mathematical model for the study and design of parabolic dish collectors with cavity receiver for use with Stirling engines, and a methodology to analyze the influence of geometric parameters, operative and climate variables on the efficiency and thermal capacity of the collection system. Additionally, in order to provide useful information for design purposes, the influence of the total error of the collector σ_{tot} on the performance of the collection system is discussed and a graphical method for design, obtained through the results generated by the numerical simulator is showed.

2. System description

The studied system consists on a parabolic dish concentrator with a cavity receiver and a Stirling engine with a 2.5 kW electric capacity. Fig. 1 shows the schematics of the solar concentrator, the cavity receiver and main parameters considered in the geometrical-optical analysis. The cavity containing the absorber is cylindrical and its aperture coincides with the focal point of the concentrator, which is located at a distance p from the rim of the parabolic dish. The absorber is located within the cavity at a distance from the aperture at which the solar image fit the entire area of the absorber.

In this study it was selected a directly illuminated tube absorber, due to its operational simplicity and the good devel-

Table 1. Design parameters and operating conditions used for the case study.

Operative conditions	
Direct solar radiation	900 W/m ²
Absorber temperature	929 K
Air speed outside the cavity	3 m/s
Ambient temperature	315 K
Absorber	
Number of tubes	32
Outside diameter of tube	0.63 cm
Material	SS 316
Area of absorber	6.44x10 ⁻² m ²
Engine	
Average pressure	2.9 MPa
Working fluid	Nitrogen
Angular speed	28 rad/seg
Swept volume	1.133x10 ⁻³ m ³
Concentrator	
Dish aperture diameter	4 m
Reflectance	0.92
Rim angle	45°
Interception factor	0.986
Focal length	2.41 m
Ratio f/D _{dish}	0.6
Total error of solar collector	8 mrad
Cavity receiver	
Cavity outside diameter	0.42 m
Aperture diameter of cavity	0.135 m
Inclination angle	40°
Insulation thickness	0.07 m
Insulation conductivity	0.042 W/(m K)
Absorptance of inner cavity surfaces	0.975
Distance from the absorber to the aperture of cavity	0.08

opment of heat transfer process when using the working fluid at high pressures and speeds [12].

Table 1 presents the operating conditions, dimensions and climate variables used to evaluate numerically the thermal performance of the cavity receiver in the case study. The operating conditions were set up considering the weather of Mexicali, B.C., México and with the purpose of operate the engine with nitrogen at low pressures, the engine parameters were established based on a previous study by the authors [13].

3. Methodology

To perform a parametric study of the concentrator and the cavity receiver, a mathematical model and an algorithm for the solution is proposed, and a numerical simulator is created. The aim of this study is to sizing the solar collection system that provides thermal energy and temperature required by the Stirling engine, under the conditions which have a minimum of optical and thermal losses.

Fig. 2 shows the diagram of the computational algorithm used to solve the mathematical model and exemplifies the exploration of the aperture diameter of the cavity, ranging

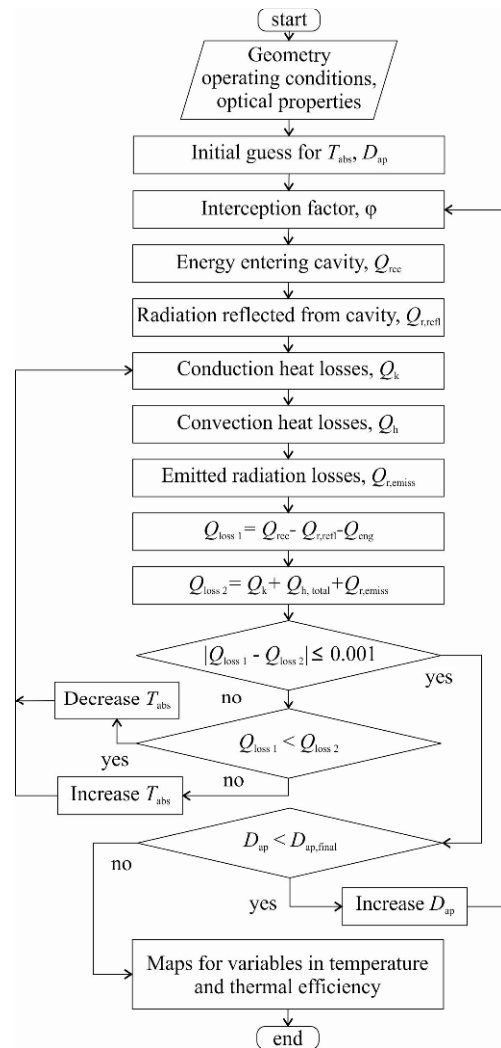


Fig. 2. Flowchart for the solution of the mathematical model.

from an initial value D_{ap} to a final value $D_{ap,final}$.

The design procedure of the solar collection system (dish-cavity) starts with a geometrical-optical performance evaluation of the concentrator. Aperture diameter and the rim angle of the parabolic dish are the variables evaluated in the operative study and the sizing of the concentrator. The rim angle is set by achieving the greatest geometric concentration and the aperture diameter of the dish is varied until meet the energy required by the Stirling engine.

Once the rim angle and aperture diameter of the parabolic dish is established, the focal distance is calculated, and with that, the geometry of the concentrator is fully defined.

Once the geometry of the concentrator is defined, the aperture diameter of the cavity D_{ap} is equal to the width of the solar image produced in the focal plane, as shown schematically in Fig. 1. Then varying the distance between the focal plane and the absorber, matching the solar image on the absorber area, the position of the absorber within the cavity is established. Finally, the aperture diameter of the cavity is varied looking for the best compromise between optical and

thermal losses, which result in maximum global efficiency. After establishing the geometry of both the concentrator and the receiver, the influence of climate variables on the operating performance of the system is analyzed and its resize depending on the involvement of these variables on system performance is considered.

For the analysis of the cavity, the energy losses included were those occurring due to natural and forced convection, conduction through the insulation and radiation both reflected and emitted. The model assumes a uniform solar energy distribution and the optical and thermal properties for cavity and absorber are kept constant. The transport properties of the air and nitrogen were evaluated using the correlations proposed by Ref. [21].

3.1 Geometric-optical model

The relationship between focal length with the rim angle and diameter of the parabolic dish is given by:

$$f = \frac{D_{\text{dish}}}{4 \tan(\psi/2)}. \quad (1)$$

The solar image width w produced by the collector in the focal zone is given by Stine and Harrigan [4]:

$$w = 2 p \tan\left(\frac{n \sigma_{\text{total}}}{2}\right) / \cos \psi \quad (2)$$

where σ_{total} is the total error of the concentrator, ψ is the rim angle, p is the distance from the concentrator surface to the focal point of the aperture and n is the number of standard deviations for the radiation beam entering the aperture based on the aperture diameter. The total error combines errors associated with physical structure, sun tracking sensor, tracking-drive system, positioning of the receiver at the focal zone and the specular reflectance values which were proposed by Stine and Harrigan [4] as typical error values for Stirling dish collector system. The optical behavior of the device is constantly affected by the error derived from the mechanics of the tracking system; for this analysis, this type of error was assumed to remain constant.

$$\sigma_{\text{total}} = \sqrt{(2\sigma_{\text{structure}})^2 + \sigma_{\text{sensor}}^2 + \sigma_{\text{drive}}^2 + \sigma_{\text{alignmen}}^2 + (2\sigma_{\text{refle}})^2 + \sigma_{\text{sun}}^2} \quad (3)$$

The intercept factor φ represents the fraction of the radiation intercepted by the cavity receiver and is calculated by Eq. (4) reported by Stine and Harrigan [4], with a change over the term of the fraction of the captured flux, using $\Gamma^{1.975}$ instead of Γ .

$$\varphi = \frac{\sum_{\psi=0}^{\psi} \Gamma^{1.975} 8\pi I f \sin(\psi) \Delta\psi}{I A_{\text{dish}}} \quad (4)$$

The shadow produced by the receiver was considered as dead area on the projected area of the dish reflective surface A_{dish} . Power consumption for the tracking and controlling systems were neglected.

3.2 Thermal model

The heat transferred to the Stirling engine is estimated by:

$$Q_{\text{engine}} = Q_{\text{rec}} - Q_{\text{r,refl}} - (Q_k + Q_h + Q_{\text{r,emiss}}) \quad (5)$$

where the beam radiation Q_{rec} entering the cavity receiver is calculated by:

$$Q_{\text{rec}} = I A_{\text{dish}} \rho \varphi. \quad (6)$$

The radiation reflected back from the cavity receiver is calculated as Duffie and Beckman [6]:

$$Q_{\text{r,refl}} = (1 - \alpha_{\text{eff}}) Q_{\text{rec}} \quad (7)$$

where α_{eff} is the effective absorbance of the cavity and is defined as:

$$\alpha_{\text{eff}} = \frac{\alpha_{\text{rec}}}{\alpha_{\text{rec}} + (1 - \alpha_{\text{rec}})(A_{\text{ap}}/A_{\text{rec}})}. \quad (8)$$

The conduction heat losses through the insulation are dissipated by convection can be calculated by:

$$Q_k = \frac{T_{\text{abs}} - T_{\text{amb}}}{\frac{L_{\text{ins}}}{k_{\text{ins}} A_{\text{rec}}} + \frac{1}{h_{\text{ext,rec}} A_{\text{rec}}}}. \quad (9)$$

The coefficient of heat transfer by convection to the outside of the cavity receiver $h_{\text{ext,rec}}$ is derived from the correlation proposed by Morgan [20]:

$$h_{\text{ext,rec}} = 0.148 (\text{Re})^{0.633} \frac{k_{\text{amb}}}{D_{\text{rec}}}. \quad (10)$$

To calculate the heat losses by natural convection (no wind), the Stine and McDonald [14] correlation was used.

$$Nu_{\text{h,nat}} = 0.088 Gr^{1/3} (T_{\text{abs}}/T_{\text{amb}})^{0.18} \cos(\theta)^{2.47} (D_{\text{ap}}/L_{\text{rec}})^s \quad (11)$$

where the Nusselt number $Nu_{\text{h,nat}}$ is based on the receiver internal diameter at cylindrical region L_{rec} , and s is defined as:

$$s = 1.12 - 0.982 (D_{\text{ap}}/L_{\text{rec}}) \quad (12)$$

and the coefficient of heat transfer by natural convection inside the cavity is calculated by:

$$h_{\text{nat}} = \frac{Nu_{h,\text{nat}} k_{\text{rec}}}{D_{\text{rec}}} \quad (13)$$

The total area for which heat losses by convection occur, corresponds to the circular area from the bottom of the cavity where the absorber is located and the area of the inner walls.

The model proposed by Ma [15] is used to calculate the convective heat losses due to wind speed considering the inclination of the receiver.

$$h_{\text{wind}} = f(\theta) \nu^{1.401} \quad (14)$$

where ν is wind speed and $f(\theta)$ is a function of the inclination angle θ and is given by:

$$f(\theta) = 0.1634 + 0.7498 \sin(\theta) - 0.5026 \sin(2\theta) + 0.3278 \sin(3\theta) \quad (15)$$

Assuming that natural convection is independent from convection due to wind, the total convection coefficient is represented by the sum of convection coefficients [15] as presented in Eq. (16).

$$h_{\text{total}} = h_{\text{nat}} + h_{\text{wind}} \quad (16)$$

The total convection heat losses from the inner walls of the cavity can be represented by:

$$Q_{h,\text{total}} = h_{\text{total}} A_{\text{rec}} (T_{\text{abs}} - T_{\text{amb}}) \quad (17)$$

and the radiation losses by emission are calculated with the model proposed by Incropera and DeWitt [22]:

$$Q_{r,\text{emiss}} = \epsilon_{\text{rec}} \sigma A_{\text{ap}} (T_{\text{abs}}^4 - T_{\text{amb}}^4) \quad (18)$$

where A_{ap} is a function of the aperture diameter of the cavity. From the general energy balance on the receiver, the total energy losses can be calculated as:

$$Q_{\text{loss}} = Q_{\text{rec}} - Q_{r,\text{refl}} - Q_{\text{engine}} \quad (19)$$

It also can be represented by summing all the contributions to the total heat loss:

$$Q_{\text{loss}} = Q_k + Q_{h,\text{nat}} + Q_{h,\text{wind}} + Q_{r,\text{emiss}} \quad (20)$$

Parabolic dish efficiency η_{dish} is equivalent to the reflectance of the surface of the concentrator ρ and the thermal efficiency of the cavity receiver is defined as the heat transferred to the Stirling engine divided by the solar energy reflected by the parabolic dish.

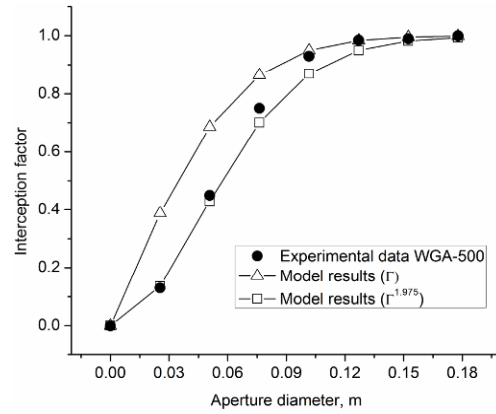


Fig. 3. Influence of the aperture diameter on the interception factor.

$$n_{\text{rec}} = \frac{Q_{\text{engine}}}{I A_{\text{dish}} \rho} \quad (21)$$

The efficiency of solar collection system (dish/cavity) η_{col} is defined as the power absorbed by the Stirling engine divided by the solar power falling on the aperture area of the parabolic dish.

$$n_{\text{col}} = n_{\text{dish}} n_{\text{rec}} \quad (22)$$

The global system efficiency is defined as the mechanical power produced by the engine E over the power falling on the aperture area of the parabolic dish.

$$n_{\text{global}} = \frac{E}{I A_{\text{dish}}} \quad (23)$$

4. Numerical model validation

In order to validate the model the results obtained with the numerical simulation were compared with theoretical and experimental results reported in literature, introducing their same geometrical configuration and operating conditions.

In the comparisons made, the representativeness of the model with respect to others from the literature [5, 11, 15–17] is evaluated by varying the angle of inclination, the aperture diameter of the cavity, wind speed and temperature of the absorber on the optical and thermal losses of the cavity receiver.

Fig. 3 shows the comparison of the interception factor that was calculated using the model described earlier and the experimental values obtained with a WGA-500 concentrator [16]. The results obtained by Eq. (4) without modification of the variable Γ show a maximum error of 177%, an average error of 29.2% and less than 1% error when the intercept factor is greater than 0.96.

However, by changing the term of the fraction of flux captured by $\Gamma^{1.975}$, the maximum error is reduced to 6.6%, the average error to 3.5% and the error is less than 2.2% when the

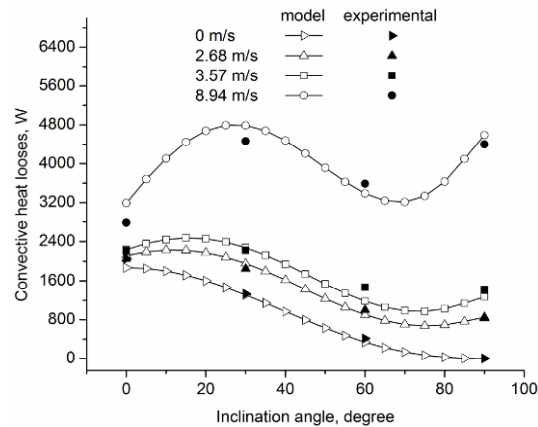


Fig. 4. Convective heat losses as function of the inclination angle of the receiver at several wind speeds. Experimental results reported by Ma [15].

intercept factor is greater than 0.96. This setting was used in obtaining results. The value by which changes the fraction of flux intercepted can be varied to further reduce the deviation of the model in a specific range on the experimental intercept factor.

Similarly, the interception factor for the collector JPL Tested with a diameter of 11 m, a rim angle of 45° , a total error of 8.54 mrad, and a cavity receiver whose aperture was 0.381 m reported by Moynihan [17] was 0.978, while the calculated with the model was 0.987 with an error of 0.92%.

Fig. 4 presents the good correlation between the simulated and the experimental results obtained by Ma [15] for the convective energy losses of the cavity receiver as a function of its angle of inclination at several wind speeds. The results simulated for the same receiver showed an average error of $\pm 7.2\%$ for the losses by convection and a maximum error of 19.6%. In support to this comparison, the correlation of Stine and McDonald [14] has been used by Nepveu et al. [3] showing consistent results; it also has been recognized as an efficient model by Sendhil and Kumar [18]. Convection heat losses vary due to the movement of the cavity produced by the solar tracking. When wind is not blowing, convective heat losses reach a maximum value when the orientation of the receiver is horizontal (aperture is facing sideways) and is minimum when the aperture is facing downwards. In the case of a fixed-focus concentrator (fixed cavity), the results of the model can be used to establish the position with less heat loss.

Fig. 5 shows the comparison of results obtained with the proposed model against the experimental results reported by Ma [15] and the theoretical results presented by Seo et al. [5].

The results reported by Ma [15] for the heat losses by radiation emission, show an excellent correlation to Eq. (18) of this model. The average error between the simulated and the experimental data was $\pm 0.5\%$ within the range explored. Subsequently, a good consistency was verified when comparing the simulated results with those reported by Seo et al. [5] except in the reflection of radiation losses, which maintained a con-

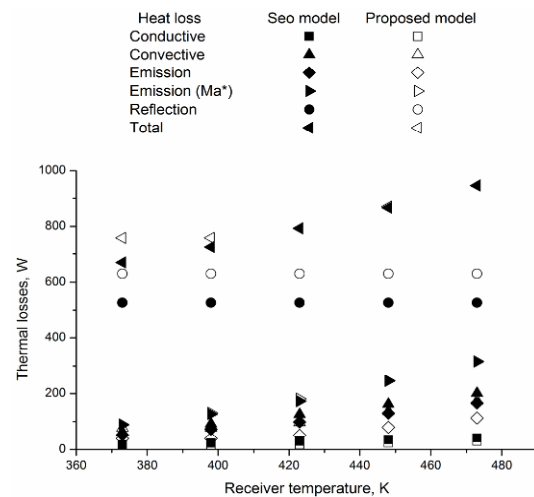


Fig. 5. Comparison for different thermal losses reported by Seo et al. [5] and emitted radiation reported by Ma [15] vs proposed model (* experimental results).

stant 100 W difference between the results of the proposed model and the results reported by Seo et al. [5]. This difference is mainly due to the fact that this model assumes that the optical properties of the cavity surfaces are uniform and remain constant regardless of temperature, while the model used by Seo, the inner surfaces of the receptor were divided into several sections where each one has its respective temperature and properties.

However as the temperature increases, the model show a better corresponding with the total energy losses. Negligible differences occur at temperature greater than 400 K.

Table 2 show the comparison of results obtained from the simulation and from the experiments carried out by Reinalter et al. [11] using a 10 kW-CNRS/PROMES.

The heat losses by radiation reflected from the receiver reached an error of 12.9%, this error may be because the real effective absorbance of the cavity is actually a composed property since two different materials are involved. The temperature calculated for the absorber showed an error of 1.7%, for the calculation of the emitted radiation was 13.7%, and for the convective heat losses was 2.8%. The error obtained for the interception factor was less than 1%.

5. Case study results and discussion

This section offers a parametric study where the dimensions of the solar collection system (dish/cavity) are defined according to the methodology described above. The main objective is to get the dimensions of the solar collection system that would meet the requirements of temperature and heat transfer rate to the Stirling engine, in conditions where the minimum optical and thermal losses are obtained. Weather conditions, operating variables, dimensions of the components and properties of materials used in the case study are shown in Table 1. The angle of inclination of the cavity is established

Table 2. Comparison of the energy losses reported by Reinalter et al. [11] and the losses obtained with the proposed model.

	Experimental		Model	
	value	uncertainty	value	% error
A	48018	±1.5%	48016	0.0
B	44400	±1.9%	44414	0.0
C	37750	±3.1%	37780	0.1
D	1400	±25%	1606.7	12.9
E	2590	±16%	2278.9	13.7
F	1000	±25%	1029.1	2.8
G	1130	±50%	1130	0.0
H	1123		1104	1.7

A – Solar energy (W)

E – Emitted radiation (W)

B – Concentrated radiation (W)

F – Convective heat losses² (W)

C – Intercepted radiation¹ (W)

G – Stirling package loss³ (W)

D – Reflected radiation (W)

H – Absorber temperature⁴ (K)

¹ Considering a total error of 6.52 mrad

² With a cavity inclination of 21° and a wind speed of 1 m/s

³ Has described by Reinalter et al. [11]

⁴ Establishing a temperature of 423 K for cavity walls³

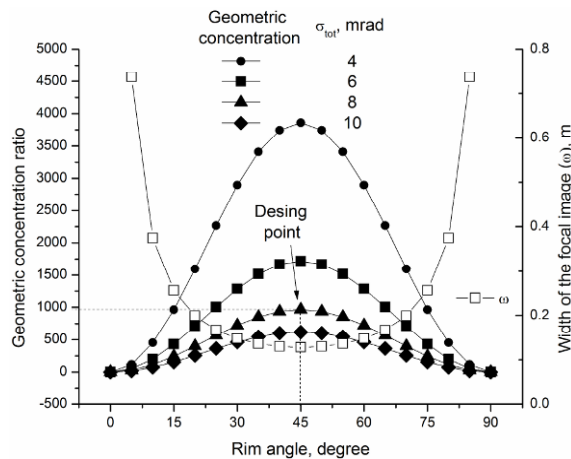


Fig. 6. Influence of the rim angle of the dish on the geometric concentration $A_{\text{dish}}/A_{\text{ap}}$ and the width of the solar image in the focal plane corresponding to a total of 8 mrad.

considering the average position of the receiver during the day.

The materials and thicknesses are established in order to operate safely and efficiently at the temperatures and pressures used in the Stirling engine, while weather conditions were established considering the city of Mexicali, BC, Mexico.

Fig. 6 shows the optical-geometrical study made to the parabolic dish concentrator and can be seen that as the rim angle of the parabolic dish increases, a geometric point of maximum concentration can be obtained, which corresponds to the minimum width of the solar image. The highest geometric con-

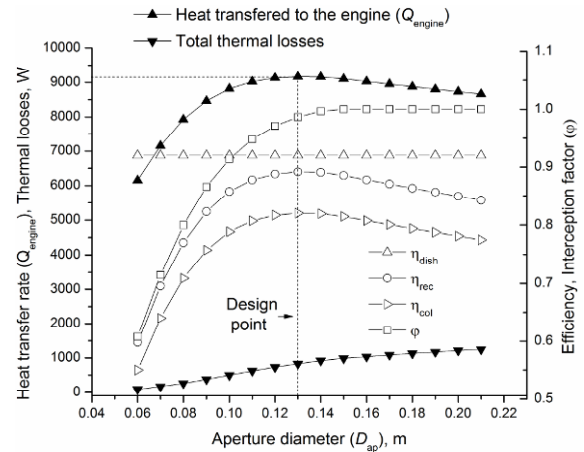


Fig. 7. Influence of the aperture diameter of the cavity on the thermal performance of the system.

centration on a receiver plane (cavity aperture) occurs at an rim angle of 45°, regardless of the total error collector, because it achieves the best compromise between the width of the solar image caused by the angle incidence of the reflected rays to the plane of the cavity aperture and the width of the image because of the separation between the focal point and the surface of the parabolic dish. Considering a rim angle of 45° and varying the diameter of the parabolic dish to satisfy the heat transfer rate required by the engine (9130 W) gives an aperture diameter of 4 m. The focal length of the parabolic dish obtained from Eq. (1) results in 2.41 m. Once the rim angle of the parabolic dish is defined and knowing the width of the absorber (0.2 m) of the Stirling engine, the solar image is matched to the size of the absorber, which results in a cavity depth of 0.08 m. Subsequently the aperture diameter of the cavity is explored, seeking to achieve the best compromise between intercepted energy and thermal losses of the cavity receiver.

The Fig. 7 allows finding the cavity aperture diameter for which the maximum heat transfer rate to the Stirling engine is obtained, where the aperture diameter has a direct relationship with the interception factor, finding that the optimum occurs for values close to unity. It is important to highlight that the aperture diameter has a considerable influence on the total thermal losses. It is noted that as the aperture diameter of the cavity reaches a point of maximum efficiency and heat transfer to the Stirling engine when the aperture diameter is equal to 0.13 m.

The heat transfer and efficiency of the receiver increases while the aperture diameter is increased due that the interception of radiation reflected by the parabolic dish is increasing, taking a greater effect on the heat losses, whereas after the maximum point has the opposite effect. With the above, all geometric parameters of the solar collection system are totally defined.

Once established the dimensions of the solar collection system is of great importance to study the thermal behavior and

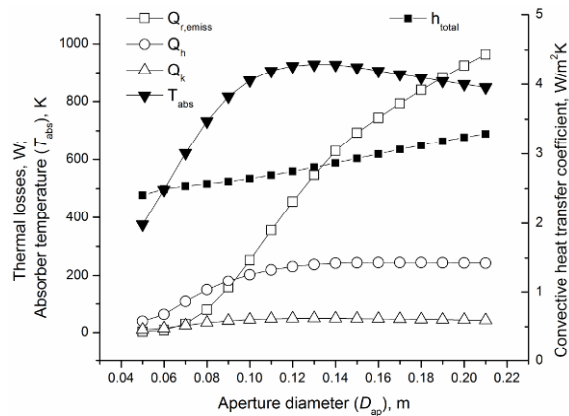


Fig. 8. Influence of the aperture diameter of the cavity on the thermal losses.

the influence of climate variables on the system capacity and efficiency.

The thermal behavior of the system is shown in Fig. 8, where can be observed that as the aperture diameter of the cavity increases, reaches a maximum temperature of the absorber where this point coincides with the maximum efficiency of the receiver in a diameter of 0.13 m. After this point, the temperature of the absorber decreases because the effect of emission of radiation losses is greater than the amount of solar radiation intercepted.

It was found that the design point of aperture diameter of the cavity corresponds to an interception factor of less than 1 (0.986), this is because the effect of heat loss is greater than the benefit to intercept the entire radiation reflected by the parabolic dish. Also, can be observed that convection losses are increasing and remain constant after aperture diameter of 0.13 m. Analyzing the terms of Eq. (17) can be appreciated that this behavior is because as the absorber temperature begins to drop (see Fig. 8), the coefficient of heat transfer by convection increases, offsetting the effect of decrease in temperature, keeping the heat losses by convection practically constant since the absorber area is unchanged.

Fig. 9 shows the influence of solar radiation on the efficiency of the receiver and the heat transferred to the engine. As expected, can be observed that the increase in radiation directly influences the temperature of the absorber and the amount of heat transferred to the motor. However, with the increasing radiation, the efficiency of the receiver decreases, due to that increasing the absorber temperature increases thermal losses. When the aperture diameter of the cavity is set to 0.13 m, an almost optimal behavior is achieved when radiation varies from 700 W/m² to 1000 W/m². It is important to note that the aperture diameter of the cavity where it achieves the highest efficiency and capacity decreases slightly with increasing solar radiation and temperature of the absorber. For systems working at temperatures under 670 K, this phenomenon occurs to a lesser extent and will be of lower importance seeking the balance point between energy intercepted and

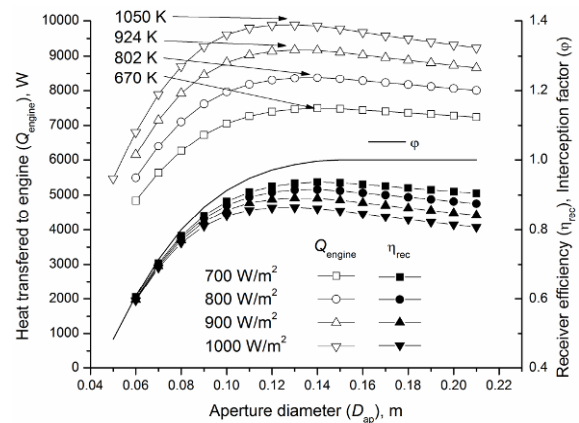


Fig. 9. Influence of radiation and aperture diameter on heat transferred to Stirling engine and receiver efficiency. The temperatures of the absorber at maximum power points are indicated.

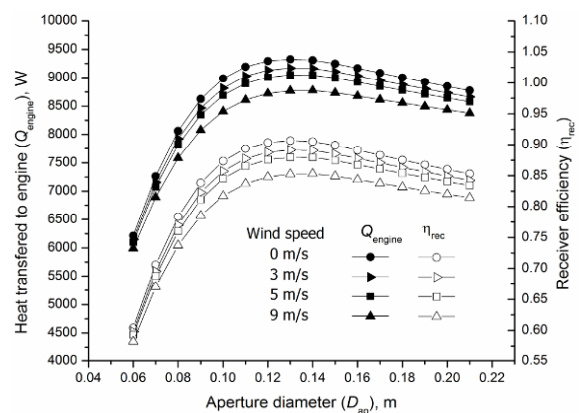


Fig. 10. Influence of the wind velocity on the heat transfer rate to the engine Q_{engine} and the receiver efficiency.

thermal losses through the aperture.

Fig. 10 shows the influence of wind speed on the heat transfer rate to the engine Q_{eng} and the efficiency of the receiver. With increasing wind speed reduces heat transfer rate to the engine and the thermal efficiency of the receiver, due to increased thermal losses. However the design point of aperture diameter of the cavity for which maximum efficiency of the receiver is achieved remains constant.

Fig. 11 shows the influence of ambient temperature on the efficiency of the receiver, the heat transferred to the Stirling engine and the temperature of the absorber. It is highlighted from the results that as the ambient temperature decreases, the heat transfer rate supplied to the engine and the efficiency of the receiver increases. This is because with decreasing temperature, the engine more easily dissipates waste heat, which lowers the temperature of the engine working gas and can remove a greater amount of heat in the absorber. By increasing the amount of heat transferred to the engine, the temperature of the absorber decreases, which causes a decrease in heat losses and increased the efficiency of the receiver. The de-

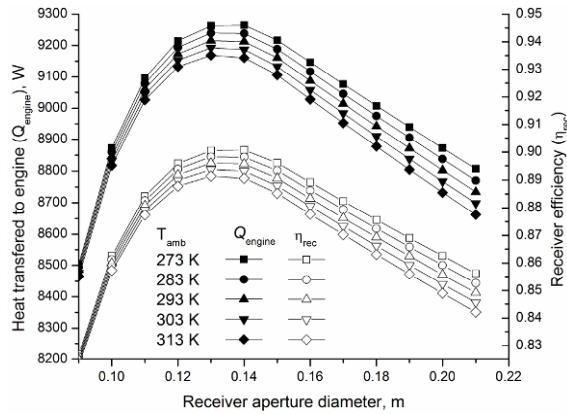


Fig. 11. Influence of the ambient temperature on the heat transfer rate to the engine and the thermal efficiency of the receiver.

crease in temperature of the absorber causes the opposite effect to that presented by reduced ambient temperature on heat loss, but the effect of lowering the temperature of the absorber is predominant.

6. General analysis and graphic sizing method

This section offers a general analysis of the influence of different geometric parameters including the total error of the concentrator on the absorber temperature, efficiency of the receiver and heat transfer rate to the Stirling engine, as well as the sizing of the case study using a graphic method, which was generated from the results obtained with the simulator.

With the support of Fig. 12 can be find the maximum global efficiency and its corresponding relationship D_{ap}/D_{dish} and absorber temperature, knowing the total error of the collector σ_{total} and solar radiation. Analyzing Fig. 12 is important to note that the highest temperatures and global efficiencies are achieved with high accuracy solar collection systems (lows σ_{total}), while the effect on solar radiation on the relationship D_{ap}/D_{dish} is more significant for solar collection systems with total optical errors of greater magnitude.

Fig. 13 is presented in order to obtain the maximum efficiency of the cavity receiver, once defined the total error of the collector and solar radiation for design. The results shown indicate that the increase in total error of the solar collector and the solar radiation, negatively influence on the receiver efficiency.

As an example, following is the sizing of the solar collection system of the case study using only Figs. 12 and 13. Whereas solar resource is known, the reflectance of the surface and the total optical error of the concentrator (which is directly related to the manufacturing method), as well as the aperture diameter of the parabolic dish which depends on the amount of heat required by the Stirling engine, with the support of Fig. 12 gives the global efficiency, the absorber temperature and the relationship D_{ap}/D_{dish} , where D_{ap} is obtained.

For the case study is assumed that the available solar radiation is 900 W/m^2 and that the total optical error of the collector

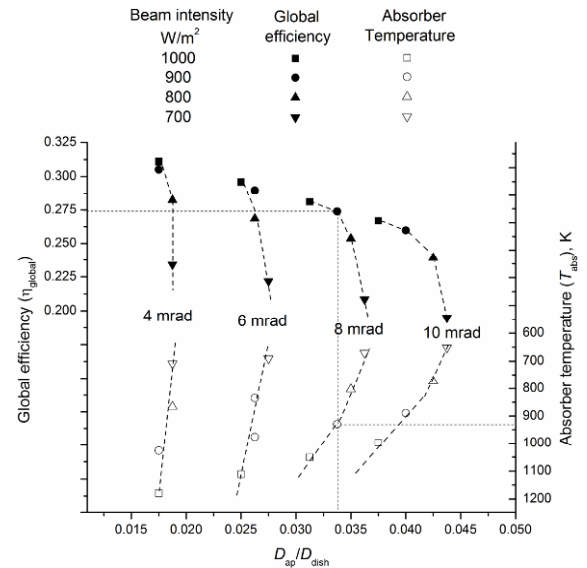


Fig. 12. Influence of the D_{ap}/D_{dish} ratio on the maximum global efficiency and the temperature of the absorber, considering different solar radiation intensities and different total errors of concentrator.

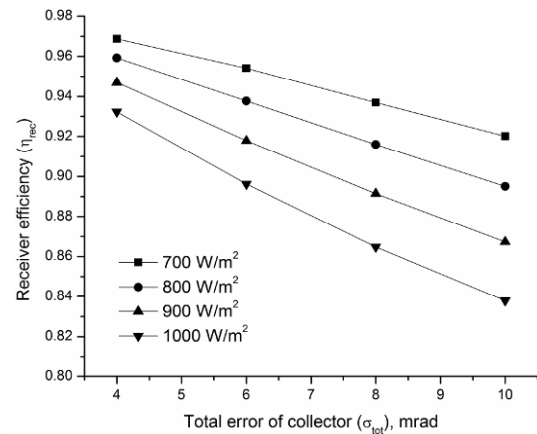


Fig. 13. Influence of the total error of the collector and radiation on the maximum efficiency of the receiver.

is 8 mrad.

By intercepting these two variables in Fig. 12, it is found a ratio of 0.03375. Since the diameter of the concentrator is 4 m, the optimal cavity aperture diameter is 0.135; working at an absorber temperature of 930 K with a global efficiency of 27.5%. The average heat transferred to the Stirling engine, will be calculated through the efficiency of the receiver read out in Fig. 13, where using the same total error of the collector and solar radiation, an efficiency of 89% is obtained, and by using Eq. (21) a 9130W heat transferred to the engine is calculated.

To design a more efficient collector system (dish/cavity) and considering that it is not possible to change the solar resource availability at which is designed, it appears that the only way is reducing the total error of the concentrator, look-

ing to reduce the D_{ap}/D_{dish} relationship. The establishment of a new design point looking for greater efficiency must also consider the increase of the energy produced by the engine and the economics associated with manufacturing more precise concentration systems.

7. Conclusions

A mathematical model was proposed and a case study of a parabolic dish concentrator with cavity receiver for a Stirling engine was designed and analyzed. The results of the simulation were compared with experimental and theoretical results reported in the literature, finding that the model predicts adequately the optical and thermal behavior of the described system, so that the model can be used to study the operation and also to design any of its components. Through the proposed mathematical model, an easy and quick graphic method was developed for sizing of dish-cavity systems.

The highest concentration on a receiver plane (cavity aperture) occurs at a rim angle of 45° , regardless of the total optical error of the solar collector. Considering the compromise between energy intercepted and thermal losses of the cavity receiver, with the proposed methodology is established for a case study, that the best aperture diameter of the cavity corresponds to 0.13 m, with an intercept factor of 0.98, thus enabling the highest heat transfer rate to the Stirling engine. With the technology available today, the maximum temperature reached in the absorber of a dish/cavity, is limited mainly by the emission of radiation losses, so it is recommended to optimize the aperture diameter of the cavity trying to reduce such losses. The increase in wind speed negatively affects the efficiency of the receiver, but does not affect the optimal diameter of the cavity aperture. The optimum aperture diameter of the cavity is less affected by the variation of solar radiation as the total error of the collector decreases. With increasing solar radiation, there is an increase in heat transferred to the engine; however the efficiency of the receiver decreases, due to increased thermal losses, so it is suggested to obtain the maximum global system efficiency, achieving a compromise between the heat transfer rate to the engine and the efficiency of the receiver.

The analysis of the case study did present a useful heat of 9130 W with an operating temperature of 930 K and a global efficiency of 27.5%. The search for greater thermal efficiency must include economic issues related to the manufacture of more precise concentration systems.

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Nomenclature

A	: Area, m^2
D	: Diameter, m
E	: Engine power, W
f	: Focal length, m
$f(\theta)$	: Parameter of Eq. (14)
Gr	: Grashof number
h	: Convective heat transfer coefficient, W/m^2K
I	: Direct solar radiation, W/m^2
k	: Thermal conductivity, W/mK
L	: Thickness, length, m
n	: Number of standard deviations
Nu	: Nusselt number
p	: Distance from concentrator surface to focal point, m
Q	: Heat transfer rate, W
Re	: Reynolds number
T	: Temperature, K
v	: Wind speed, m/s
w	: Width of the focal image, m

Greek symbols

α	: Absorbance
$\Delta\psi$	: Rim angle increment
ε	: Emissivity
σ	: Stefan-Boltzman constant, $W/m^2 \cdot K^4$
σ_{total}	: Total error of collector, mrad
η	: Efficiency
ρ	: Surface reflectance
φ	: Interception factor
θ	: Inclination angle of the cavity, degree
ψ	: Rim angle, degree
Γ	: Flux capture fraction

Subindex

<i>abs</i>	: Absorber
<i>amb</i>	: Ambient
<i>ap</i>	: Cavity aperture
<i>col</i>	: Collector
<i>dish</i>	: Dish collector
<i>eff</i>	: Effective
<i>engine</i>	: Stirling engine
<i>ext</i>	: Exterior
<i>global</i>	: Solar to mechanical power conversion
<i>h</i>	: Convection
<i>ins</i>	: Insulation
<i>k</i>	: Conduction, conductivity
<i>loss</i>	: Energy losses
<i>nat</i>	: Natural
<i>rec</i>	: Cavity receiver
<i>r, emiss</i>	: Radiation emitted
<i>r, refle</i>	: Radiation reflected
<i>s</i>	: Parameter of Eq. (11)

wind : Wind speed

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Ricardo Beltran received his B. Eng. degree in Mechanical Engineering from Universidad Autónoma de Baja California, México in 2005. He then received his Ph.D degree in Engineering from Centro de Estudio de las Energías Renovables, UABC in 2011. He is currently a Subject professor at Facultad de Ingeniería at Universidad Autónoma de Baja California. His research interest includes solar power, Stirling engines and energy storage.



Nicolas Velazquez received his M. S. degree in Engineering from Instituto Tecnológico de Celaya, México. He then received his Ph.D degree in Engineering from Facultad de Química and Centro de Investigación en Energía at Universidad Nacional Autónoma de México. He is currently a Research professor and head of the Centro de Estudio de las Energías Renovables at Instituto de Ingeniería of the Universidad Autónoma de Baja California. His research interest includes solar heating, advanced solar cooling.